Improved Multi-Axial, Temperature and Time Dependent (MATT) Failure Model

D. E. Richardson, G. L. Anderson, and D. J. Macon
ATK Thiokol Propulsion Corp.
Brigham City, UT
e-mail: david.richardson@atk.com

Introduction

An extensive effort has recently been completed by the Space Shuttle's Reusable Solid Rocket Motor (RSRM) nozzle program to completely characterize the effects of multi-axial loading, temperature and time on the failure characteristics of three filled epoxy adhesives (TIGA 321, EA913NA, EA946). As part of this effort, a single general failure criterion was developed that accounted for these effects simultaneously. This model was named the "Multi-Axial, Temperature, and Time Dependent" or MATT failure criterion[1,2,3].

Due to the intricate nature of the failure criterion, some parameters were required to be calculated using complex equations or numerical methods. This paper documents some simple but accurate modifications to the failure criterion to allow for calculations of failure conditions without complex equations or numerical techniques.

Theoretical

The following Multi-Axial Temperature and Time (MATT) dependent failure model was originally proposed[1,2,3].

$$AP^2J_2 + BPI_1 = 1 \tag{1}$$

Here J_2 is the second deviatoric stress invariant, and I_1 is the first stress invariant. A and B are shape parameters that define the ellipsoidal nature of the failure envelope. These parameters have been shown to be independent of temperature or time.

For a constant P value, this failure criterion is equivalent to the Tsai-Wu[4] failure model and equivalent to a modified Drucker-Prager failure model[5].

P is a scaling factor that scales the failure envelope to a proper level for a given temperature and failure time. This factor is found using a linear cumulative damage model[6] approach. The linear cumulative damage failure model has the following form.

$$N_\sigma = \left[\int_0^{t_f} \sigma_i^\beta \, dt\right]^{1/\beta} \tag{2}$$

Here N_σ and β are experimentally determined failure parameters, σ_i is an applied stress as a function of time with a failure time t_f. The linear cumulative damage equation can be simplified for the following basic loading conditions.

For constant loading rate conditions:

$$t_f = (1+\beta)\left[\frac{N_\sigma}{\sigma_f}\right]^\beta \qquad \sigma_f = N_\sigma\left[\frac{t_f}{1+\beta}\right]^{\frac{-1}{\beta}} \tag{3}$$

For constant stress (or creep) loading conditions:

$$t_f = \left(\frac{N_\sigma}{\sigma_f}\right)^\beta \qquad \sigma_f = N_\sigma t_f^{\frac{-1}{\beta}} \tag{4}$$

Using these simple relationships and normalizing the A and B parameters, the MATT failure criterion can be modified to the following form for constant loading rate evaluations:

$$A_\sigma B_\sigma^{\,2}\left(\frac{t_f}{1+\beta}\right)^{\frac{2}{\beta}} J_2 + B_\sigma\left(\frac{t_f}{1+\beta}\right)^{\frac{1}{\beta}} I_1 = 1 \tag{5}$$

For constant load studies:

$$A_\sigma B_\sigma^{\,2} t_f^{\frac{2}{\beta}} J_2 + B_\sigma t_f^{\frac{1}{\beta}} I_1 = 1 \tag{6}$$

Here the B_σ takes on additional meaning because it becomes a combined MATT failure parameter. B_σ contributes to definition of both shape and size of the failure ellipse (compare with equation 1). The A_σ parameter noted in this equation can be different than that used in equation 1. Here, A_σ is a shape parameter that is specifically normalized to the linear cumulative damage term seen in equations 5 and 6 (A in equation 1 can be normalized to anything).

For the materials evaluated in this paper, it was determined that the B_σ and the β parameters are a linear function of temperature:

$$\beta = m_\beta T + b_\beta \tag{7}$$

$$B_\sigma = m_N T + b_N \tag{8}$$

Here the m_β, b_β, m_N, and b_N terms are the traditional slope and intercept parameters, and T is the temperature.

Calculations of failure can be obtained by substituting the results of equations 7 and 8 into equations 5 and 6. As will be shown in subsequent sections, failure for a wide range of multi-axial, temperature, and time conditions can be defined by the MATT equation using only five coefficients (A_σ, m_β, b_β, m_N, b_N).

Experimental

Extensive test data were used to characterize failure of TIGA 321 and EA946, and limited test data were used to characterize failure of EA913NA. As will be seen, even with limited characterization data, accurate failure models are developed. Testing was conducted below the glass transition temperature for TIGA 321 and EA913NA. For EA946, testing was conducted in the glass transition regime. The results of this characterization are of particular interest due to this low glass-transition temperature.

For this study, the time and temperature dependent nature of the adhesives was characterized using tensile adhesion test specimens. These tests were used to determine the m_β, b_β, m_N, and b_N terms. Shear adhesion tests were used to characterize the effects of multi-axial loading on failure. From these tests, the A_σ parameter was obtained. Tests were conducted under temperature conditions that ranged from (20 °C to 45 °C), with failure times that ranged from several minutes to several hours, and with pure tension or pure shear.

Verification of the accuracy of the failure model was evaluated using napkin ring test specimens and creep loading of tensile adhesion buttons. The materials were tested under temperature conditions that ranged from (20 °C to 45 °C), with failure times that ranged from several minutes to several months, and with a wide range of multi-axial loading (tension/compression combined with shear).

Results and Discussion

The results of tensile and shear adhesion tests can be seen in the Figures 1-3 for the adhesives TIGA 321 and EA946. These figures include both the raw test data and the MATT predictions of failure. Each data point is an average of several tests (the number varies from 6 to 16 depending on the condition, the adhesive, and the test). The coefficients of variation for these tests are in Table 1.

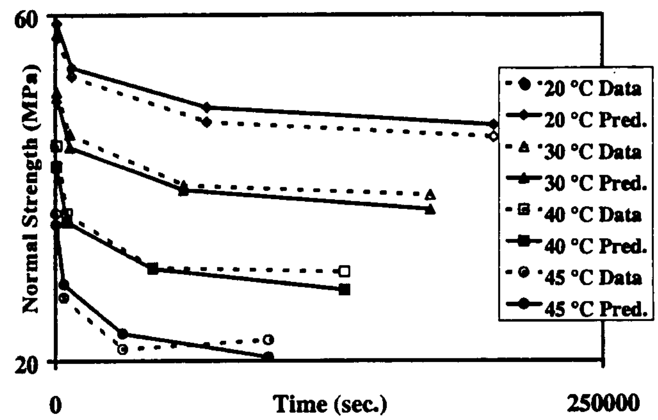

Figure 1. TIGA 321 Tensile Adhesion Data /Predictions

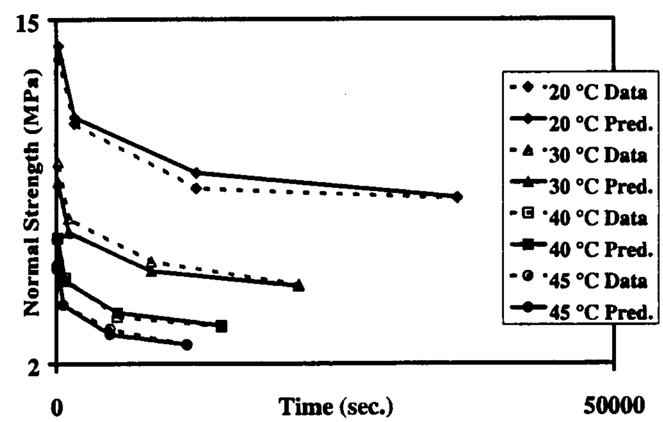

Figure 2. EA946 Tensile Adhesion Data /Predictions

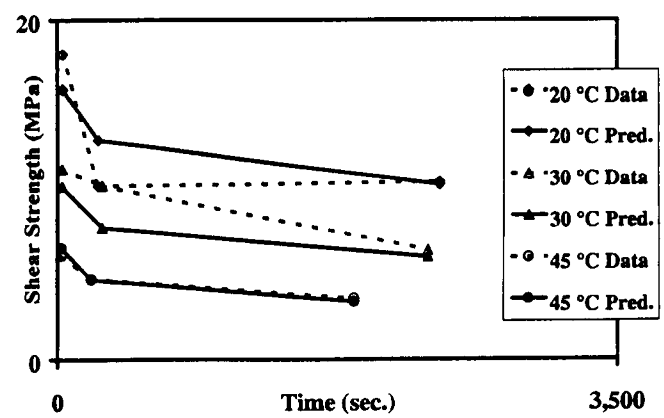

Figure 3. EA946 Shear Adhesion Data /Predictions

Table 1. Coefficients of Variation for the Tensile Adhesion and Shear Adhesion Tests

Adhesive	Tensile	Shear
TIGA 321	9%	11%
EA913NA	7%	7%
EA946	10%	22%

Figures 5-8 show the multi-axial and creep test data and predictions that were used for verification of the material model for TIGA 321 and EA946. Each data point represents the average of 2 to 13 data points. Table 2 contains the coefficients of variation for this testing

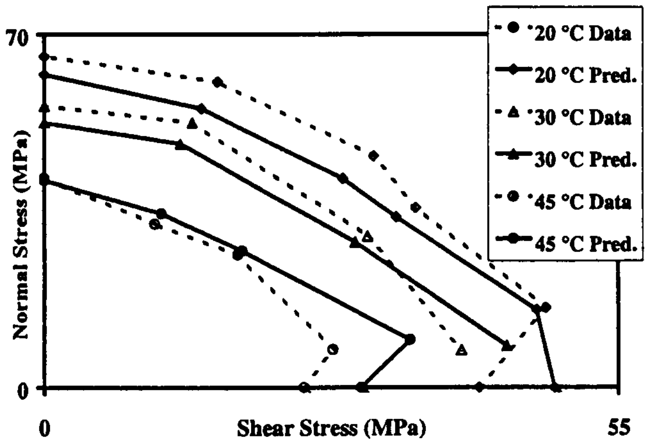

Figure 4. TIGA 321 Multi-Axial Data /Predictions

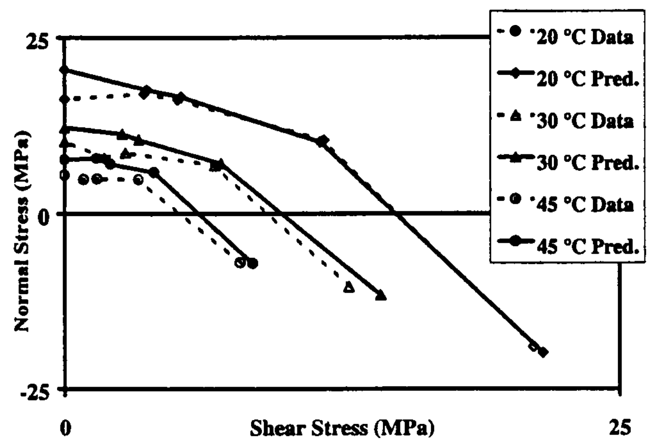

Figure 5. EA946 Multi-Axial Data /Predictions

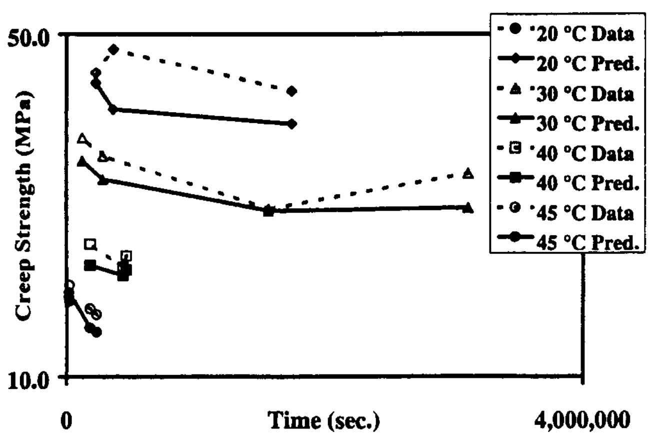

Figure 6. TIGA 321 Creep Data /Predictions

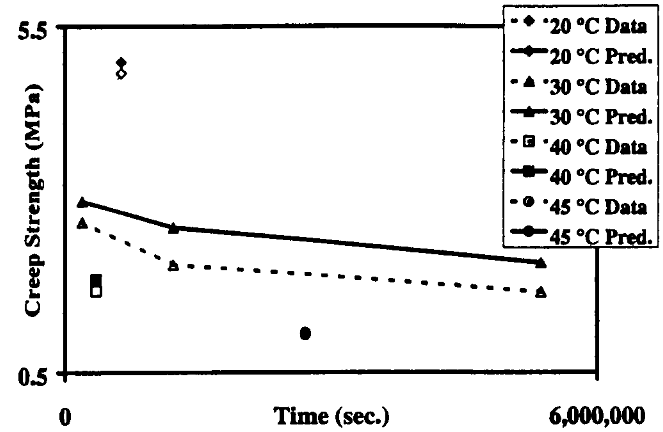

Figure 7. EA946 Creep Data /Predictions

Table 2. Coefficients of Variation for the Multi-Axial and Creep Tests

Adhesive	Multi-Axial	Creep
TIGA 321	13%	15%
EA913NA	11%	21%
EA946	23%	20%

Conclusions

The improved MATT failure criterion has been shown to be accurate for a wide range of conditions. Coefficients of variation for all the data combined are seen in Table 3.

Table 3. Coefficients of Variation for All Tests

TIGA 321	EA913NA	EA946
11%	9%	17%

Of particular interest is the accuracy of the model for the adhesive EA946. Failure for this adhesive is characterized and the model is verified for conditions that pass through the glass transition of the adhesive. The test data indicate the generality of the failure criterion for a wide range of materials (even through the glass transition).

References

1. D. E. Richardson, M. L. McLennan, G. L. Anderson, D. J Macon, and A. Batista-Rodriguez, *Proc. of the 25th Annual Meeting of The Adhesion Society, Inc.*, 2002, pp. 68-70.
2. D. E. Richardson, M. L. McLennan, G. L. Anderson, D. J Macon, and A. Batista-Rodriguez, *J. Adhesion*, (in press).
3. D. E. Richardson, G. L. Anderson, and D. J Macon, *Proceedings of the AIAA JPC*, 2002.
4. S. W. Tsai and E. M. Wu, *J. Compos. Mater.*, 1971, 5, pp. 58-80.
5. D. C. Drucker and W. Prager, *Q. Appl. Math.*, 1952, 10, pp. 157-165.
6. K. L. Laheru, *J. Propuls. Power*, 1992, 8, pp. 756-759.

THE ADHESION SOCIETY COPYRIGHT FORM

This signed form, appropriately completed, must accompany any abstract published by the Adhesion Society.

Title of Paper:

Authors:

Publication Title: ***Proceedings of the 26th Annual Meeting of the Adhesion Society***

Publication Date: ***February 2002***

Return Form To:
Esther Brann
2110 Hahn Hall – 0344
Blacksburg, VA, USA
24061
FAX: 540-231-8517

Part A – COPYRIGHT TRANSFER FORM

The undersigned, desiring to publish the above paper in a publication of the Adhesion Society, hereby transfer their copyrights in the above paper to the Adhesion Society, Inc., also known as the Adhesion Society.

Returned rights. In return for these rights, the Adhesion Society hereby grants to the above authors, and the employers for whom this work was performed, royalty-free permission to:

1. Retain all proprietary rights other than copyright, such as patent rights.

2. Resuse all or portions of the above paper in other works.

3. Reproduce, or have reproduced, the above paper for the author's personal use or for company use provided that a) the source and the Adhesion Society's copyright are indicated, b) the copies are not used in a way that implies Adhesion Society endorsement of a product or service of an employer, and c) the copies *per se* are not offered for sale.

4. Make limited distribution of all or portions of the above paper prior to publication.

5. In the case of work performed under U.S. Government contract, The Adhesion Society grants the U.S. Government royalty-free permission to reproduce all or portions of the above paper, and to authorize others to do so, for U.S. Government purposes. ***(U.S. Government employees complete Part B)***

Adhesion Society obligations: In exercising its rights under copyright, the Adhesion Society will make all reasonable efforts to act in the interests of the authors and employers as well as in its own interest. In particular, the Adhesion Society requires that:

1. The consent of the first-named author be sought as a condition in granting re-publication permission to others.

2. The consent of the undersigned employer be obtained as a condition in granting permission to others to reuse all or portions of the paper for promotion or marketing purposes.

In the event the above paper is not accepted and published by the Adhesion Society or is withdrawn by the author(s) before acceptance by the Adhesion Society, this agreement becomes null and void.

ATK Thiokol

Authorized Signature | *Employer for Whom Work Was Done*

11-15-02

Title if not Author | *Date Form Signed*

PART B (to be completed by U.S. Government Employees)

This will certify that all authors of the above paper are employees of the U.S. Government and perform this work as part of their employment and that the paper is therefore not subject to U.S. copyright protection. If the work was performed under government contract and you are not a government employee, complete PART A.

Authorized Signature | *Employer for Whom Work Was Done*

Title if not Author | *Date Form Signed*